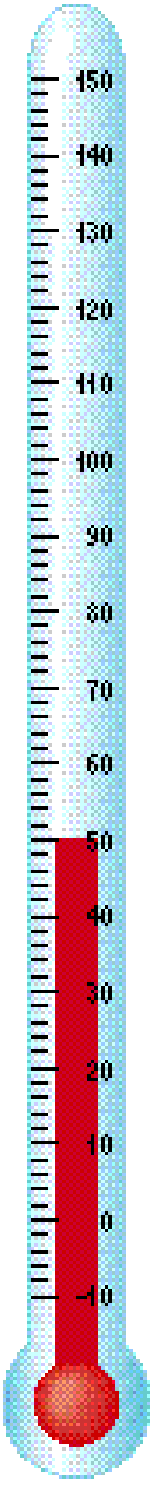
Section 3

Central Tendency

The primary concept underlying this section is to explore ways to represent and analyze temperature data.

The activities involve students in collecting and analyzing temperatures in the classroom. Students determine how much the classroom temperature varies at a given time; they decide what measure they think is best to represent the typical temperature in the room; and they examine the way in which extreme data points affect the measures of central tendency students are exploring.

7 Is Our Room All One Temperature?



How much does room temperature vary in the classroom? In this activity, students make conjectures about the variability of the temperature in the classroom. They decide where they think temperatures are high and where they think temperatures are low. Students use analog thermometers to test their hypotheses and record the temperatures they find.

math goals

- Students read analog thermometers.
- Students examine the concept of calibration (equating scales so that all instruments read the same at the same location and the same time).
- Students make conjectures about room temperature.
- Students make bar graphs.
- Students consider maximum, minimum, and range of temperature readings in the room.

ongoing assessment

This activity will provide many opportunities for you to assess students' ability to read and interpret temperatures and make sense of the data they collect.

advance preparation

materials

12 or more analog thermometers

tape or glue stick

Reproducible Masters 6 (Celsius Thermometer), 7 (Fahrenheit Thermometer), and 13 (The Temperature in Our Room)

1 sheet of large chart paper or poster board

what students do

1. Students make conjectures about the temperature in the classroom.

   Ask students to consider questions like these:
   - What is the temperature in our room?
   - Is the temperature likely to be the same in every location in the room?
   - How can we decide what our room temperature is?

2. Students calibrate the thermometers to be sure they all read the same temperature in the same location.

   Encourage students to consider why the thermometers need to be calibrated and to decide how to deal with differences in temperature readings from thermometers in the same location.

3. Students make conjectures about where in the classroom the extremes of temperature exist.

   List students' suggestions on the board. If students suggest that certain areas of the room will be warmer or cooler than others, ask them to explain their reasoning. Then ask whether other students agree, and if so, why or why not.

4. Using the class consensus about the locations of highs and lows, students place the thermometers around the room.

   Explain that each group will be responsible for a certain number of thermometers.

5. Students record the temperatures and construct a "thermometer bar graph" of the data.

   Depending on the scale students are reading, have them use Reproducible

Teachers'

Teacher A

The day before we did this activity, I put all 10 of the thermometers inside my top desk drawer to be sure they all recorded the same temperature. As it turned out, one of the thermometers registered 2 degrees Fahrenheit under the others. I decided to let the students ponder this difference.

The next day, after introducing the activity, I mentioned the notion of calibrating the thermometers. Students all agreed that in order to get accurate data, the thermometers had to read the same temperature while in the same location. I asked each group to determine how it would deal with the "off" thermometer. A few groups decided to discard the data from that thermometer—they were not convinced that it would always be under by exactly 2 degrees. The other groups decided to adjust the reading of that thermometer by adding 2 degrees Fahrenheit.

Teacher B

When it came time to place the thermometers around the room, my students came up with many good suggestions. For example:

- Skye suggested we place one on top of the cubbies, because he thought it seemed cooler in that part of the room.
- Stephanie wanted a thermometer on a desk in the middle of the room, to see if that would give the typical temperature.
- Carl thought near the plants on the teacher's desk might be different.

One student suggested taping a thermometer to a window. We had a

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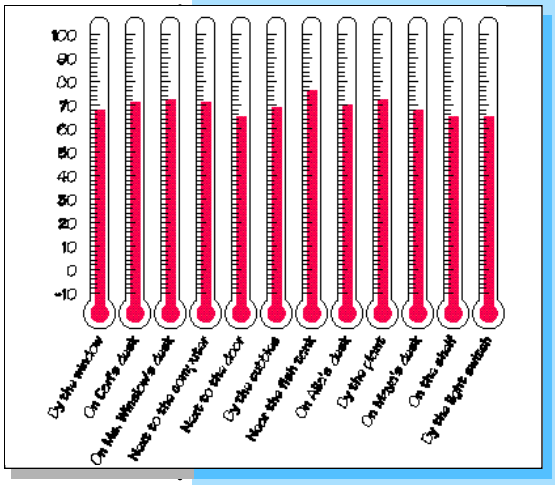
lengthy discussion about whether or not that location was "fair." Some students felt that the temperature on the window should not be considered the temperature in the room. Others felt that the window was part of the room and wanted to include a thermometer on the window. A few students also felt that having a thermometer on the window would help us get a wide range of temperatures.

The students finally reached a compromise by deciding to place a thermometer on the sill under the window, rather than taping it right on the window.

Teacher C

My students wanted to be systematic about placing the thermometers around the room. We had 12 thermometers. Students placed 6 on desks—2 toward the front, 2 in the middle, and 2 in the back. Then they placed the other 6 around the perimeter of the room, paying more attention to the distance between the thermometers than to their exact location.

After we chose a place for each thermometer, Mina volunteered to be the timekeeper. At the designated time, she called out "Time!" Each group recorded the temperature from one thermometer and colored in the temperature in both Celsius and Fahrenheit on the handout. At right is the graph we constructed.



Master 6 (Celsius Thermometer) or 7 (Fahrenheit Thermomter) to label and color pictures of thermometers representing the actual temperature readings. Then have students construct a class bar graph by cutting out their thermometer pictures and arranging them on an axis. Encourage students to arrange the thermometer pictures in ways that help them see patterns. Students may, for instance, choose to put the pictures on the graph in ascending or descending order; they may also add axes and a scale to their bar graphs

6. Students discuss the data.

   Ask students questions like the following:
   - What is the range of temperatures?
   - Were you surprised that the range is so large / small?
   - Was the warmest / coldest place where you expected it to be?
   - Are there other locations you want to check?
   - Do you think the temperature changes evenly between the highest point in the room and the lowest? Why or why not?
   - Should any readings be ruled out?

7. Students share the temperature data from their thermometer and record all of the data on Reproducible Master 13 (The Temperature in Our Room).

background information

Thermometers

About a dozen thermometers that measure Fahrenheit and/or Celsius will be needed and must be calibrated to see if they all record the same value for the same location and time. This calibration is one of the challenges to the class.

Calibration

To collect accurate temperature data, all of the thermometers must show the same temperature when they are in the same place. *To calibrate* means to check, adjust, or systematically standardize the graduations of a quantitative measuring instrument. *To calibrate* can also mean to determine or mark the graduations on a measuring instrument.

In this activity, the concern is that the thermometers register the same temperature for the same location. As a class, take some time to check the thermometers. For the purposes of this activity, students should either (1) calibrate their thermometers by discarding ones that deviate substantially from the norm, (2) adjust the placement of the tube in the backing so that it reads the same as the preponderance of other thermometers, or (3) figure out how the thermometer deviates and adjust the results when it is read.

Variations in Temperature

Your classroom may have a temperature variation of 5 degrees or more, depending on local conditions (e.g., an air-conditioning or heating vent in one corner of the room; a spot that gets sun most of the day).

Students will have many good ideas about where to place the thermometers around the room to get a wide range of temperatures. To see how students are thinking about the problem, ask each student to give a reason as well as a suggestion for where to place a thermometer.

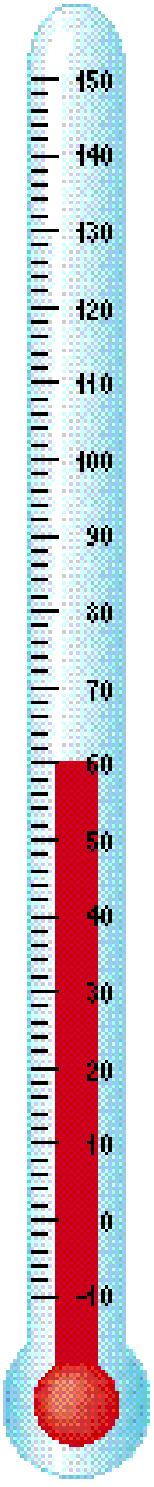
The range of the data—the difference between the highest and the lowest data points—is another way to examine variation.

Bar Graphs

There are many ways to graphically represent data; this activity uses bar graphs. The advantages of the bar graph for data representation of room temperature are as follows:

- Differences in data are readily visible.
- Common data points are easy to pick out.
- The method provides a quick “picture” of data that would otherwise be difficult to see.

8 How Can We Describe Our Room Temperature?



What would students say if someone asked, "What's the temperature in your classroom?" In this activity, students try to decide what their "typical" room temperature is. Students then examine more formal methods of computing central tendency and decide whether or not these are appropriate to describe their room temperature.

math goals

- Students examine the concept of "typical," or average, and explain their reasoning.
- Students are introduced to measures of central tendency: mode, median, and mean.
- Students discuss which measure of central tendency best describes the typica" temperature in the room and in a given situation.
- Students explore the effects of extreme data points on the mode, median, and mean of a data set.

ongoing assessment

The number students choose to represent the room temperature and the explanations they give for their choice will provide you with insights into the understanding they already have about how to summarize a set of data.

advance preparation

materials

room temperature data from Activity 7

what students do

1. Using the numeric data from Activity 7, students choose one number that best represents the "typical" temperature in the classroom and explain why they chose that number.

   Direct students to discuss the temperature readings and graphs within their groups and decide what temperature they think best represents their room temperature. After they have finished, ask the groups to present to the class their idea of room temperature and an explanation of why they chose it.

   This is an informal way of addressing the concept of average, or most typical.

2. Students think of ways that an average temperature for their city or town could be described.

   Students may suggest that the readings for the town could be averaged. Try to get at students' ideas of what an average is. How many temperature readings do they think would be necessary? Are there places in town that they think should definitely be included or excluded in the average?

3. Students are introduced to the three types of averages: mode, median, and mean.

   Be sure students discuss the meanings of each of these and determine which are relevant and accurate based on the data and the context.

Teachers'

Teacher A

I started by posing the question "If you had to report the temperature in the classroom, what number would you choose?" I asked each group to work on this question and prepare an explanation of its choice.

The data:

65° 66° 66° 67° 68° 68°
69° 72° 72° 72°

Here is a summary of the groups' reports:

Group	Temp.	Explanation
A	72°	Three out of 10 thermometers read 72. This was the most common temperature.
B	69°	This was the temperature in the middle of the room.
C	68.5°	This is halfway between the highest and the lowest temperature.
D	68.5°	This is the "average." (Added all 10 temperatures and divided by 10.)
E	68°	This is the middle temperature.
F	69°	This is the temperature around the middle of the room.

After the group reports, I asked for the class's comments.

Darell said, "I don't think 72 is a good choice, because most of the data are under 70."

Then I asked Group A to explain its thinking.

"We just picked 72 because it happened more than the others and because the 72s were all near the middle of the room," Simira explained.

Stories

The class continued to argue about which temperature was the most accurate representation of our room temperature.

Then Marielle asked, "Why did Group C get 68.5 and Group E get 68? Didn't they both do the same thing?"

We discussed the difference between finding the halfway point between the highest and lowest and taking the *middl*e temperature (the median).

I explained that 65 was the lowest temperature and 72 was the highest. Halfway between those two is 68.5, which is the same as taking the mean of the highest and lowest. Then, I added that there are 10 temperatures. The two middle temperatures are 68, so the median is 68.

This was the perfect entry into a lesson on mode, median, and mean!

Teacher B

Our class discussion about the average temperature in the room was making some students unhappy because it seemed like there was no "right answer." It all seemed to depend on where you put the thermometers, how many you used, when you read them, and whether you used a mode, median, or mean to determine the average. Then Shawna said something that really helped us think about the problem differently. She said if we could stir up all the air in the room like mixing up various colors of paint, all of the temperatures would turn into one temperature, and that would be the average. It wouldn't matter if we used the mean, the median, or the mode—they would all be the same.

4. Students describe situations where one type of average would be more appropriate than the others.

   Suggest widely different sets of data and ask students how they would use the various measures of central tendency to describe them. For example, to some students mode may seem meaningful only in cases where there is an overwhelming set of numbers that are all the same. Other students might suggest that mean would be most useful in a situation where there is no mode, unless a few temperatures are very different from the rest, in which case median would be the best choice.

5. Students decide whether they wish to revise their typical temperatures and / or collect more temperature data.

6. Students consider what they think would happen to the mode, median, and mean temperatures if, for example, one of the thermometers in the room was placed in front of an air conditioner.

   Have students recalculate their measures of central tendency with a very different (high or low) temperature included in the set. Ask them:

   - How does this extreme data point affect your calculations?
   - What would happen if there were more data points in the set? Fewer data points?

background information

Following are temperature data collected from one classroom at 9:00 AM on a warm sunny day. This class used 27 readings.

65	66	67	68	68	68	68	68	68	68
68	69	69	69	69	70	70	71	71	71
72	72	73	74	74	97	99			

Mode

The mode is 68 degrees. This is the temperature that occurred most frequently. Although the mode is one type of average, some students may argue that 68 degrees is not a good representation of the temperature in this classroom, since 16 of the 27 data points are greater than 68 degrees. Other students may argue that since it occurred most often, 68 degrees is an accurate description of the typical classroom temperature. This could make for a lively and meaningful class discussion!

Median

To determine the median, first put the data in order from lowest to highest (or vice versa). The median is the *middle* data point. For the 27 data points in the above example, the median is the fourteenth data point, 69 degrees. Some students may argue that 69 degrees is an accurate representation of the classroom temperature because it is the middle of the data (i.e., half of the data fall above the median and half of the data fall below it). Others may argue that the median does not reflect the range of the data. Were there an even number of data points—say, 28—the median would be halfway between the two middle data points.

Mean

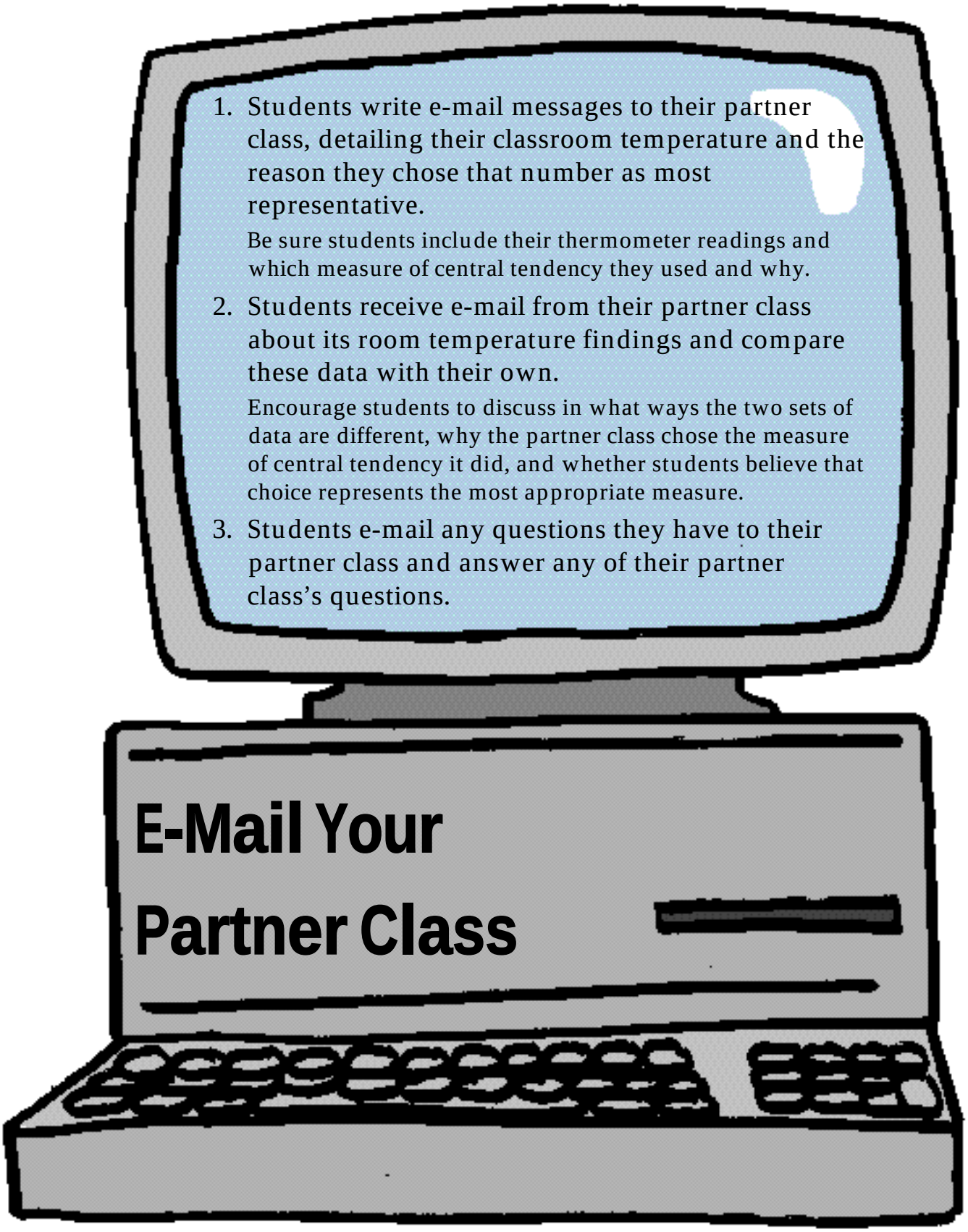
To calculate the mean temperature, add all of the data (in this example, the sum is 1,932). Then divide by the number of data points: 932 ÷ 27 = 71.6. The mean can be thought of as a way of "evening out" the data.

Some students may argue that the mean temperature, 71.6 degrees, is the best representation of the temperature in the classroom because it takes into account all of the data. Others may say that the high temperatures, 97 and 99 degrees, are unfair because they were taken in direct sunlight and cause the mean to be unrealistically high. Still others may believe that the

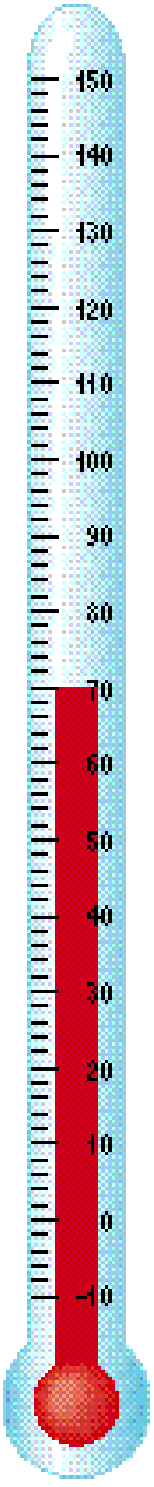
mean would be the best representation but only if there were more thermometers in the classroom.

Concept of Average

There is usually no single best way to calculate the average room temperature. Sometimes the median seems to be the most representative number; sometimes it's the mean or the mode. Students should be familiar with all three methods of determining an average and should have a chance to discuss how each one represents a similar or different measure of the typical room temperature for the data they collect.


1. Students write e-mail messages to their partner class, detailing their classroom temperature and the reason they chose that number as most representative.
Be sure students include their thermometer readings and which measure of central tendency they used and why.
2. Students receive e-mail from their partner class about its room temperature findings and compare these data with their own.
Encourage students to discuss in what ways the two sets of data are different, why the partner class chose the measure of central tendency it did, and whether students believe that choice represents the most appropriate measure.
3. Students e-mail any questions they have to their partner class and answer any of their partner class's questions.
E-Mail Your
Partner Class

9 Sampling and Comparing Temperatures



How is the average temperature for a particular place found? What quantity of data is needed? What measures are useful for describing and comparing temperatures at two different locations? Do the answers to these questions change if the temperatures are mostly alike or mostly different? In this activity, students consider such questions as they learn more about measures of central tendency in terms of temperature data at different geographic locations.

math goals

- Students discuss ways to gather data to determine the "average temperature" at a location.
- Students report on and critique their plans.
- Students are encouraged to consider how the choice of measure of central tendency is affected by sample size, placement of recording devices, timing, range, and repetition of specific readings.
- Students identify extremes, compute the range, and find the mean, median, and mode of given sets of data.
- Students make decisions based on an analysis of data.

ongoing assessment

The class discussion will inform you about students' ideas for how to collect temperature data. Reproducible Masters 14 and 15 will help you evaluate students' understanding of how to use and interpret measures of central tendency.

advance preparation

materials

1 copy of Reproducible Masters 14 (Comparing Temperatures) and 15 (Sampling Temperatures) for each pair of students

calculators

index cards for use with the sampling activity described on Reproducible Master 15 (Sampling Temperatures)

what students do

1. The class explores the question "How do you find the average temperature?"

   Each group takes one of these locations:
   - Death Valley in July
   - a pond in northern Maine on a typical winter day
   - your school between 5:00 PM and 5:00 AM
   - New York City at 6:00 AM on April 1
   - the moon
   - the place you were born, each year on your birthday

   In their groups, students talk over the answers to these questions:
   - How many thermometers would you need?
   - Where would you put them?
   - How often would you read them?
   - After collecting your data, what method would you use to determine the average temperature?

   Explain that one student in each group is to record the group's best thinking and report how this group would collect data and determine an average temperature.

2. As the groups report, students in other groups ask questions or suggest different strategies.

   Use this discussion to focus students' attention on certain points about data collection and analysis, such as these:
   - The sample size (number of readings) depends in part on the variability of the data. If the temperatures are unlikely to differ much, the sample can be smaller than if the temperatures are expected to vary widely.

Teachers'

Teacher A

This activity made me realize that statistics isn't quite the dry subject I once thought it was. I began to see how much problem solving is involved in simply figuring out how you are going to collect data. Also, together with the students I began to see that you can get quite different answers depending on how you set things up. We all came to the conclusion that you have to consider what your resources are—how much time and money you have—and what question you really want to answer. So that's why different statistics can be reported for the same thing. For example, two different groups of scientists calculated the average worldwide temperature and arrived at two different numbers. They weren't that far apart, but at least now my students and I recognize that neither is "wrong" or "right"—they're both simply very good estimates.

Stories

Teacher B

Some students, like Terry, thought that you could find the mean by averaging the two middle data points. I asked students to work out a counterexample. One of the counterexamples was this set of four data points: 67, 68, 72, 74.

We calculated the mean: 281 ÷ 4 = 70.25, and the median: (68 + 72) ÷ 2 = 70.

Terry and a few others then proposed that perhaps the mean or the median could be found more easily by averaging the highest and lowest number. We tried that on the same data set and found that (67 + 74) ÷ 2 = 70.5, which is neither the mean or the median for the data set.

I was surprised that in our work on central tendencies, my students had the most difficulty with median. The most common misconception was that finding the middle data point is the same as taking the mean of the high and low data points. We spent some extra time working with other data sets.

For homework, I asked students to create three more examples of data sets for which the mean and median were the same, and three for which they were different.

- The placement of thermometers and the timing of the readings affect the quality of the data that will be collected. Ideally, thermometers should be placed where they can capture the full range of temperatures. Similarly, readings should be taken at times that will permit a full range to be collected.
- If all of the temperatures that are collected are quite close together, then the mean, median, and possibly mode will also be close together.
- If the temperatures are spread fairly evenly over a wide range, the mean is probably best to represent the average.
- If one or two temperatures are very different from the rest, the median may be a better representation of the average.
- If an overwhelming number of temperatures are the same, the mode may be the best way to represent the average. However, if the thermometers were situated and the readings were timed only to capture the extreme temperatures, then the mode would not be an appropriate measure of the average temperature.

3. Students, equipped with calculators, work in small groups to complete the tables presented in Reproducible Masters 14 (Comparing Temperatures) and 15 (Sampling Temperatures).

   Ensure that students first discuss answers to each question as a group and then write their responses individually.

4. The class discusses the results of the sampling activity, shares the responses students wrote to the questions, and draws conclusions about the relationship between sample size and the amount of variation in a set of data.

background information

Sampling

A sample is a set of items chosen from a large population. Samples are taken in order to infer a set of characteristics for the larger population Samples are often used to examine populations that are too large to be surveyed individually. For example, wildlife biologists capture a representative number of individuals from an animal population in order to determine the number of animals in a given area, the distribution of age and gender, and health characteristics.

Section 3 Assessment

Students use the Section 3 Quiz, found at the end of the collection of Reproducible Masters, for this assessment.

Assessment Goals

The items in this assessment test students' understanding of concepts related to measuring central tendency. To perform well on this assessment, students should be able to

- estimate an appropriate temperature range, mean, median, and mode;
- take into account the effects of sample size and variability on measures of central tendency; and
- describe an appropriate method for finding an average temperature.

Answer Key

Many good answers are possible for these open-ended questions. A rubric is in development for use in evaluating student responses.

Go to Section 4