Are Spiral Galaxies Round?

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Introduction

Although astronomers have observed galaxies for hundreds of years, their true nature has only been determined in the past seven decades. In 1923, Edwin Hubble threw open the door to the universe when he took a series of photographs of the Andromeda "Nebula," and discovered that it contained Cepheid variable stars. By carefully measuring their variation in brightness over time, he was able to determine an approximate distance to these stars. These results, along with subsequent measurements, proved that this "nebula" was not only much farther away than anyone had suspected, but was larger than our own Milky Way Galaxy. When he published his results in 1924, he demonstrated that the universe was far larger and more populated than was commonly believed.

We now know that there are more galaxies in the universe than there are stars in our own galaxy. Just as astronomers have developed various ways of classifying stars, they have also developed taxonomies for galaxies. Hubble devised the first galactic classification scheme, with four classes of galaxies (Kaufman & Freedman, 1998) :

- **Spirals**, characterized by a flat disk with curving arms of stars,
- **Barred Spirals**, in which the spiral arms originate at the ends of a "bar" of stars running through the galaxy's nucleus,
- **Ellipticals**, named for their shape, have no arms, and
- **Irregulars**, which do not fit into any of the other three classes.

Are spiral galaxies really round? Put another way, do all spiral galaxies have a thin, flat disk that is circular when seen face-on? Take a look at the images of spiral galaxies shown in Figure 1. They aren't all circular—in fact, most spiral galaxies appear to be oval or perhaps spindle-shaped. How can astronomers assert that spiral galaxies are round, simply because they have arms?

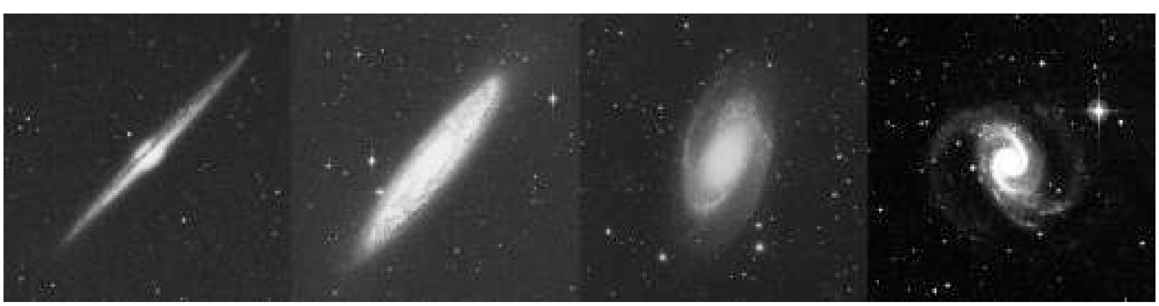


Figure 1. Images of Four Spiral Galaxies

This project demonstrates that all spiral galaxies (including barred spirals) are round (circular) flat disks, and that any apparent elongation or deformation is an artifact of the galaxy's orientation with respect to observer's line of sight. If all spiral galaxies are indeed round flat disks, a large enough sample of them should be statistically indistinguishable from a sample of randomly tilted disks.

This apparent tilting of the galactic disks can be measured simply, as will be shown in the next section, and is known as the galaxy's **inclination** (i). The rest of this project will analyze the distribution of galactic inclinations in both published catalogs as well as in a random sample of galaxies measured by the author.

Methods for determining galactic inclination

Before we analyze the distribution of galactic inclination values, they must first be determined. Fortunately, this is relatively simple to accomplish. Consider the illustration in Figure 2. Each ellipse represents a disk (circle) that is seen at a different angle. The circle represents a disk that is face-on and has an inclination of 0°. As the disk is tilted, the inclination increases until it reaches 90° as the disk is seen edge-on as a line and has an inclination of 90°.

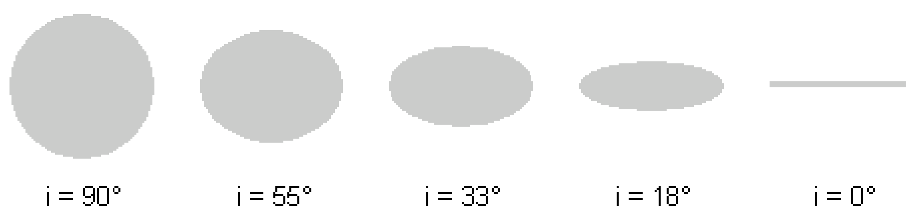


Figure 2. Tilted Disks with Inclination Values

But how is this inclination determined? The easiest method is to measure the axial ratio and apply trigonometry to determine the tilt of the galaxy.

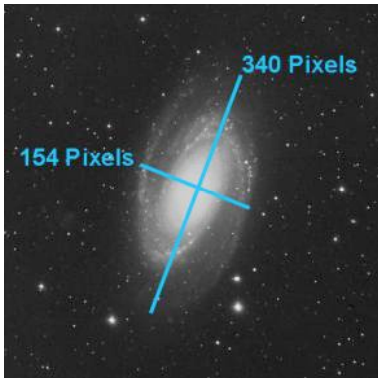
Axial ratios (simple)

One method (the one used in the illustration above) is to use the **axial ratio** of the disk. To determine the axial ratio, measure the minor axis diameter and divide it by the major axis diameter. Once you have determined the ratio, use the following formula to determine the tilt, or inclination of the disk in degrees:

$$= \cos^{-1}(q), \qquad (1)$$

where q is the axial ratio.

Example: NGC3031



Major Axis = 340 pixels

Minor Axis = 154 pixels

q = 154/340 = .4529

i = $\cos^{-1}$(.4529) = 63°

Therefore, this galaxy appears to be tilted away from us by 63°.

However, as we will see in the next section, this simple equation cannot be used to reliably determine a galaxy's inclination.

Axial Ratios (complex)

While mathematically sound for measuring the inclination of simple disks, equation (1) fails to take into account two factors that are important when measuring actual galaxies.

The first of these factors is the thickness of galactic disk. When viewing a spiral galaxy edge-on, the axial ratio will never reach 0, since all galaxies have a non-zero thickness. Therefore a correction factor must be added to correctly identify an edge-on, high-inclination spiral galaxy.

Here is the formula that Hubble used to determine galactic inclination (Hubble, 1926) :

$$\cos^2 i = (q^2 - q_o^2)/(1 - q_o^2), \qquad (2)$$

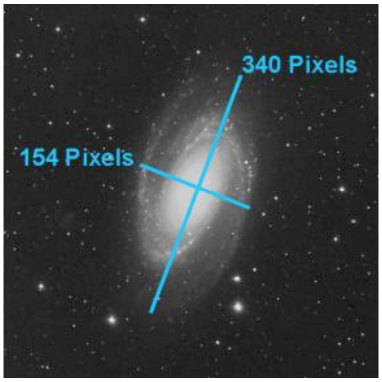
where q is the axial ratio and q_o^2 is the minimum axial ratio that should be considered to be edge-on (i=90°). The commonly accepted value for q_o^2 is 0.2. In other words, if a galaxy's disk appears more than five times longer than it is wide, we assume that the galaxy's inclination is 90° (Tully & Fisher, 1977) .

The second factor is a systemic correction that takes into account another inherent inaccuracy in measuring a galaxy's axial ratio. If you compare the inclinations calculated with axial ratios with inclinations determined by radio mapping techniques, you will discover a small but consistent discrepancy—the axial ratios indicate that galaxies are too "face-on" by a few degrees (Aaronson, Mould, & Huchra, 1980) . While this study indicated that calculated inclinations were too low by 2° (±6°), most astronomers adopt a correction of +3° (Tully, 1988) .

Therefore, the fully corrected formula for determining galactic inclination from axial ratios is:

$$i = \cos^{-1} \{(q^2 - 0.2^2)/(1 - 0.2^2)\}^{1/2} + 3° \qquad (3)$$

Example: NGC3031 (Revisited)



Major Axis = 340 pixels

Minor Axis = 154 pixels

$q = 154/340 = .4529$

$i = \cos^{-1} \{(.4529^2 - 0.2^2)/(1 - 0.2^2)\}^{1/2} + 3°$

$i = 68°$

Therefore, this galaxy is actually tilted away from us by 68°, 5° more than the “simple” formula indicated.

Analysis of published catalog values

Now that we have determined two formulas for determining galactic inclination, we will use them to determine the inclination of galaxies classified as spirals in published catalogs. used three astronomical catalogs, each of which listed galaxies by their morphological type, as well as either their axial ratio or the actual major and minor axis measurements:

1. The Nearby Galaxies Catalog (Tully, 1988), which contains 1,412 spiral galaxies,
2. The Third Reference Catalogue of Bright Galaxies (de Vaucouleurs et al., 1991), which contains 15,214 spirals, and
3. The Catalogue of Principal Galaxies (Paturel et al., 1989), which lists 33,526 spiral galaxies.

We would expect that a sample of 50,152 spiral galaxy measurements would constitute a statistically significant sample.

While it is true that each catalog contains galaxies that are found in the other catalogs, these galaxies were measured independently by each group, and should be considered separate measurements.

What distribution should we expect?

If all spiral galaxies are in fact circular disks that are randomly tilted to our line of sight, what distribution should we expect? To carefully examine the hypothesis presented in this analysis, it is necessary to be able to compare the observed distribution of galactic inclinations with the theoretically expected distribution that would obtain if all spiral galaxies were indeed round. Norris describes it this way:

"Imagine a stick being viewed at different angles.□The inclination of the stick (theta, in degrees) will be distributed uniformly, but the observed length of the stick will go as cos(theta).□You've then got two of these (theta and phi, say), and the distributions are multiplied (since any one value of theta gives you the entire range of phi). So you end up with a $\cos^2$ distribution (Norris, 2000) ."

Thus, if the observed data were to be drawn from a distribution of inclinations that were measured for a large random sample of round disks, we should observe something that is distributed $\cos^2$. Since this distribution is not generally included as a default in most statistical packages, it is necessary to create the reference distribution from scratch via simulation.□To serve as a surrogate for a cosine^2 distribution, I created a uniform rectangular sample of 500 numbers that range from 0 to 0.998.□ then calculated the cosine of each sample value and squared it.□When I plotted the results, it yielded the following curve:

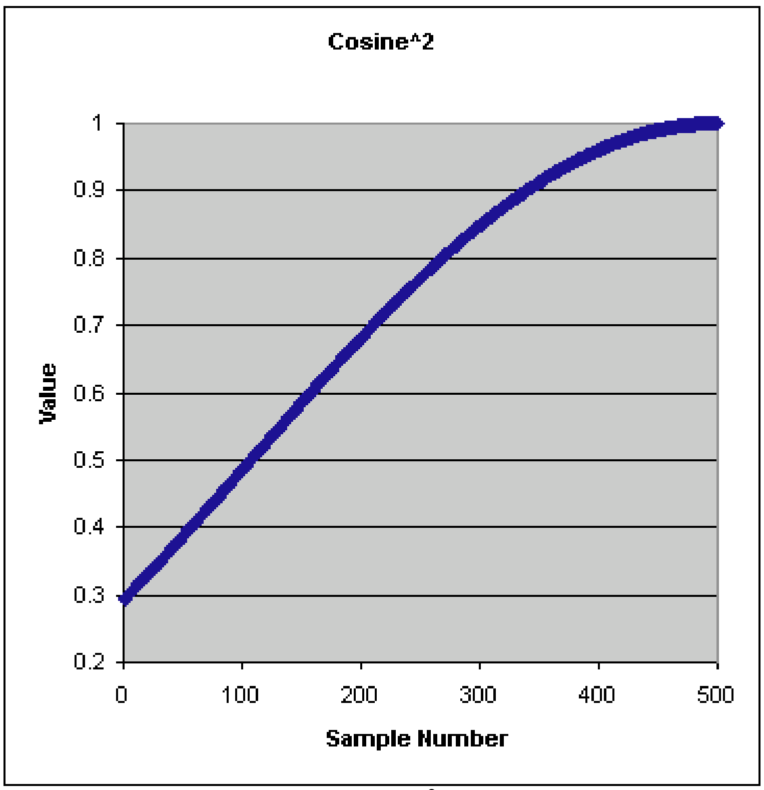


Figure 3. Reference Cosine^2 Distribution Curve

Now that we have a reference cosine^2 distribution, we can compare the inclinations determined from the published catalogs to it, and measure how closely they match.□One simple comparison of fit of two distributions (typically one an observed sample and the other a theoretical expectation) is the Kolmogorov-Smirnov test.

Analysis of "simple" calculations

In order to analyze the published catalog values for comparison with the reference cosine^2 distribution, it was first necessary to "normalize" the inclination values.□While all three catalogs provided either the axial ratio or the major and minor axis values, only the *Nearby Galaxy Catalog* provided a direct listing of inclination values.□

In order to ensure that each catalog was compared to the reference distribution correctly, the published inclination values were not used.□The axial ratio for each spiral galaxy was used (after being computed from the major and minor axis measurements, when required).□The axial ratios were first applied to the "simple" formula, and the results tabulated.

After determining the inclination values for each catalog sample, two statistical tests were applied, comparing the distribution of the computed inclination values against the reference cosine^2 distribution.□In order to facilitate the statistical analysis, I used a statistical software package—*S-PLUS 4.5* published by MathSoft, Inc.

The first test was for aggregate correlation between the catalog samples and the reference distribution.□The correlation provides a simple check to determine whether the observed data and the theoretical expectation are broadly similar.□This test returns a value bounded [-1,+1], with values closer to ±1 indicating and increasing likelihood that both the empirical observations and theoretical calculations tend to have similar (relative) values.

A more precise test was, however, undertaken to examine whether the underlying distributions of the observed and theoretical data appeared to be isomorphic. This second test was the two-sample Kolmogorov-Smirnov test .□This is a goodness-of-fit test that measures how well an "unknown" sample (in this case the catalog samples) fit the distribution of the "known" sample (the reference cosine^2 distribution).□This test returns two values: KS, which measures how well the sample fits the known curve (with 1 being a perfect fit), and p, which is the confidence value.□The higher the p-value, the more unlikely it is that the KS-value is correct.□Put another way, a p-value close to 0 indicates that there is an infinitesimally small probability that the observed and the theoretical data are drawn from *different* distributions.

So let's look at the results of these tests in the table below. □

Simple	**Nearby Galaxies Catalog** (n=1,412)	**Third Reference Catalogue of Bright Galaxies** (n=15,214)	**The Catalogue of Principal Galaxies** (n=33,526)
Correlation Values	0.9753297	.9921069	.8914653
Two-Sample Kolmogorov-Smirnov Test Values	KS = 0.9278 p = 0	KS = 1.0 p = 0	KS = 0.9017 p = 0

As you can see, the correlation values are quite high, indicating a high likelihood that they all share the same distribution. The Kolmogorov-Smirnov tests also show that each of the samples matches the reference curve fairly well, although the fit drops off as the number of samples increases.

Both of these tests are within acceptable limits, however the decreasing goodness-of-fit value is somewhat troubling.□f this decrease in fit were due to an inaccuracy in the “simple” formula, I would expect the samples to better fit the reference curve when the “complex” calculation is used.□

Analysis of “complex” calculations

The same three samples were used again, except that the “complex” formula that takes into account observational and systemic corrections.□f these corrections are correct and desirable, we should see an increase in the goodness-of –fit values.

The same two tests were performed on the results of the “complex” calculation, and are summarized below. □

Complex	**Nearby Galaxies Catalog** (n=1,412)	**Third Reference Catalogue of Bright Galaxies** (n=15,214)	**The Catalogue of Principal Galaxies** (n=33,526)
Correlation Values	0.9756108	.9920751	. .9074233
Two-Sample Kolmogorov-Smirnov Test Values	KS = 1.0 p = 0	KS = 1.0 p = 0	KS = 1.0 p = 0

Again we see that the correlations are quite high, which is exactly what is expected given the results from the “simple” calculations. The Kolmogorov-Smirnov tests, however, indicate an almost perfect correspondence exists between each sample and the distribution derived for a “round” galaxy.

These results strongly support the argument that both the observational and systematic corrections that are included in the “complex” formula are both accurate and desirable.

Now that we have used a very large sample of published measurements to determine the galactic inclination distribution, let’s measure some galaxies directly using published photographic images.

Measurement of published images

We have shown that large samples of galaxies have inclinations that closely match the reference distribution. In this section we will look at a much smaller sample and will measure the inclinations, rather than depend on published values.

Sample Selection

In order to select a random sample of spiral galaxies, I used a random number generator to produce a sample of 100 NGC galaxies that were identified as spiral types 0-9 in Tully's Nearby Galaxy Catalog. The Nearby Galaxy Catalog was chosen since all of its entries should be relatively large and bright, and therefore easy to measure.

Once I extracted the list of target galaxies, I downloaded .gif images of each galaxy from the Digital Sky Survey Web site (http://stdatu.stsci.edu/dss/). Once the images were stored on disk, I used the *Debabelizer* software package to invert each one to a negative image, since faint galactic structures are easier to see in a negative (black on white) image.

Measurement methodology

In order to calculate each galaxy's inclination, it was first necessary to measure their major and minor axis diameters. To make these measurements, I used the "Measure Tool" incorporated into Adobe's *Photoshop* application.

After loading each image, I would adjust the brightness and contrast levels to produce the most detailed version of the image. I then used the "Measure Tool" to determine the length of the major and minor axes in pixels. Since the measurements would be used only to produce a ratio, it was not necessary to determine and correct for the actual image scale. Each measurement was recorded in an Excel spreadsheet and saved for further computations.

After measuring all 100 galaxy images, I extended the spreadsheet to include inclination values determined by both the "simple" and "complex" methods. I also created a new $cosine^2$ distribution sample with 100 members, so as to provide a reference distribution for statistical testing.

Analysis of measured distribution

I performed the same two tests that were performed on the published catalog values. The results are seen below.

Measured	**100 Inclinations derived from "simple" formula**	**100 Inclinations derived from "complex" formula**
Correlation Values	0.9852001	.9870498
Two-Sample Kolmogorov-Smirnov Test Values	KS = 1.0 p = 0	KS = 1.0 p = 0

These results clearly show that the smaller sample fits the reference distribution remarkably well. The “simple” formula values actually show a better fit than the values calculated for the large samples. The “complex” formula values show the same fit (KS = 1) as the large samples.

What does this tell us?

Conclusion

We have shown that there is an almost perfect correspondence between the reference cosine^2 distribution and each of the four samples. What does this tell us about the shape of spiral galaxies?

Our hypothesis was that all spiral galaxies (including barred spirals) are round (circular) flat disks, and that any apparent elongation or deformation is an artifact of the galaxy’s orientation with respect to observer’s line of sight. If all spiral galaxies are indeed round flat disks, a large enough sample of them should be statistically indistinguishable from a sample of randomly tilted disks.

In order to test this hypothesis, we made the following assumptions:

1. All spiral galaxies are made up of a flat circular disk of stars, dust, and gas
2. Any spiral galaxy that does not appear round must be tilted from the observer’s line-on-sight
3. The distribution of inclinations resulting from tilting along two axes can be described as a cosine^2 distribution
4. A large sample of spiral galaxies should evidence the same distribution as the reference distribution

If the sample did not fit the reference distribution, it would indicate that a disk tilting along two axes couldn’t model the appearance of spiral galaxies. This would leave open the possibility that these galaxies are not all round flat disks—they might have different ellipsoidal forms, encompassing all the variations observed. This would have serious repercussions in our understanding of nature and behavior of the spiral arms.

After analyzing the inclination results of over 50,000 spiral galaxy measurements, it is apparent that the sample distribution does match the reference distribution. Therefore we can state, with a high degree of confidence, that spiral galaxies are in fact round flat disks, and that any elongation is caused by the galaxy’s orientation.

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